**Test-10**  
**Marks-15x5**

**Vector Algebra**

Q.1. Find a vector in the direction of vector \( \vec{a} = \hat{i} - 2\hat{j} \) that has magnitude 7 units.

Q.2. Show that the points A, B and C with position vectors, \( \vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k} \), \( \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \) and \( \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k} \) respectively, form the vertices of a right angled triangle.

Q.3. Find \( |\vec{a} - \vec{b}| \), if two vectors \( \vec{a} \) and \( \vec{b} \) are such that \( |\vec{a}| = 2 \), \( |\vec{b}| = 3 \) and \( \vec{a} \cdot \vec{b} = 4 \).

Q.4. Find the area of the parallelogram whose adjacent sides are determined by the vectors \( \vec{a} = \hat{i} - \hat{j} + 3\hat{k} \) and \( \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k} \).

Q.5. If a unit vector \( \vec{a} \) makes angles \( \frac{\pi}{3} \) with \( \hat{i} \), \( \frac{\pi}{4} \) with \( \hat{j} \) and acute angle \( \theta \) with \( \hat{k} \), then find \( \theta \) and hence the components of \( \vec{a} \).

Q.6. Let \( \vec{a} \), \( \vec{b} \) and \( \vec{c} \) be three vectors such that \( |\vec{a}| = 3 \), \( |\vec{b}| = 4 \), \( |\vec{c}| = 5 \) and each one of them being perpendicular to the sum of other two, Find \( |\vec{a} + \vec{b} + \vec{c}| \).

Q.7. The scalar product of the vector \( \hat{i} + \hat{j} + \hat{k} \) with a unit vector along the sum of vectors \( 2\hat{i} + 4\hat{j} - 5\hat{k} \) and \( \lambda \hat{i} + 2\hat{j} + 3\hat{k} \) is equal to one. Find the value of \( \lambda \).

Q.9. If the sum of two unit vectors is a unit vector, Prove that the magnitude of their difference is \( \sqrt{3} \).

Q.10. Show that the points A, B, C with position vectors \( -2\vec{a} + 3\vec{b} + 5\vec{c} \), \( \vec{a} + 2\vec{b} + 3\vec{c} \) and \( 7\vec{a} - \vec{c} \) respectively are collinear.

Q.11. If a vector makes \( \alpha \), \( \beta \), \( \gamma \) with OX, OY and OZ respectively, prove that \( \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2 \).

Q.12. If \( \hat{a} \) and \( \hat{b} \) are unit vectors inclined at an angle \( \theta \), then prove that \( \sin \frac{\theta}{2} = \frac{1}{2} |\hat{a} - \hat{b}| \).

Q.13. If \( \vec{a} + \vec{b} + \vec{c} = 0 \), \( |\vec{a}| = 3 \), \( |\vec{b}| = 5 \), \( |\vec{c}| = 7 \), find the angle between \( \vec{a} \) and \( \vec{b} \).

Q.14. If \( |\vec{a}| = 2 \), \( |\vec{b}| = 5 \) and \( |\vec{a} \times \vec{b}| = 8 \), find \( \vec{a} \cdot \vec{b} \).

Q.15. Determine whether the four points A(-2,0,3), B(1,0,0), C(1,-3,3) and D(4,1,-2) are coplanar.