Application of Derivatives

Q.1. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

Q.2. Use differentials to find the approximate value of $\sqrt{0.037}$.

Q.3. It is given that for the function $f(x) = x^3 - 6x^2 + ax + b$ on $[1, 3]$, Rolle’s theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of $a$ and $b$ if $f(1) = f(3) = 0$.

Q.4. Find a point on the curve $y = (x - 3)^2$, where the tangent is parallel to the line joining $(4, 1)$ and $(3, 0)$.

Q.5. Find the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is decreasing or increasing.

Q.6. Find the local maximum or local minimum of the function.
$$f(x) = \sin^4 x + \cos^4 x, \quad 0 < x < \frac{\pi}{2}.$$  

Q.7. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, 1)$.

Q.8. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

Q.9. A balloon which always remain spherical has a variable diameter $\frac{3}{2}(2x + 1)$. Find the rate of change of its volume with respect to $x$.

Q.10. Find the intervals in which $f(x) = (x+1)^3 (x - 3)^3$ is strictly increasing or decreasing.

Q.11. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Q.12. Using differentials, find the approximate value of $(26.57)^{1/3}$.

Q.13. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Q.14. Find the equation of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(x_0, y_0)$.

Q.15. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.